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Game Theory 350

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An Analysis of John Q

Game theory is a mathematical approach used to model behavior in strategic situations, often games, in which an individual's success in making decisions depends on the choices of others. By looking at player preferences, certain outcomes can be predicted. However, game theory does not only apply to what we commonly thing of as "games". It can be used to model wars, opera's, conflicts, arguments, and much more. What happens when game theory is used to model the behavior of actors in a movie? Can one figure out their preferences and strategies and predict the outcome of the movie? This is the basis for this project.

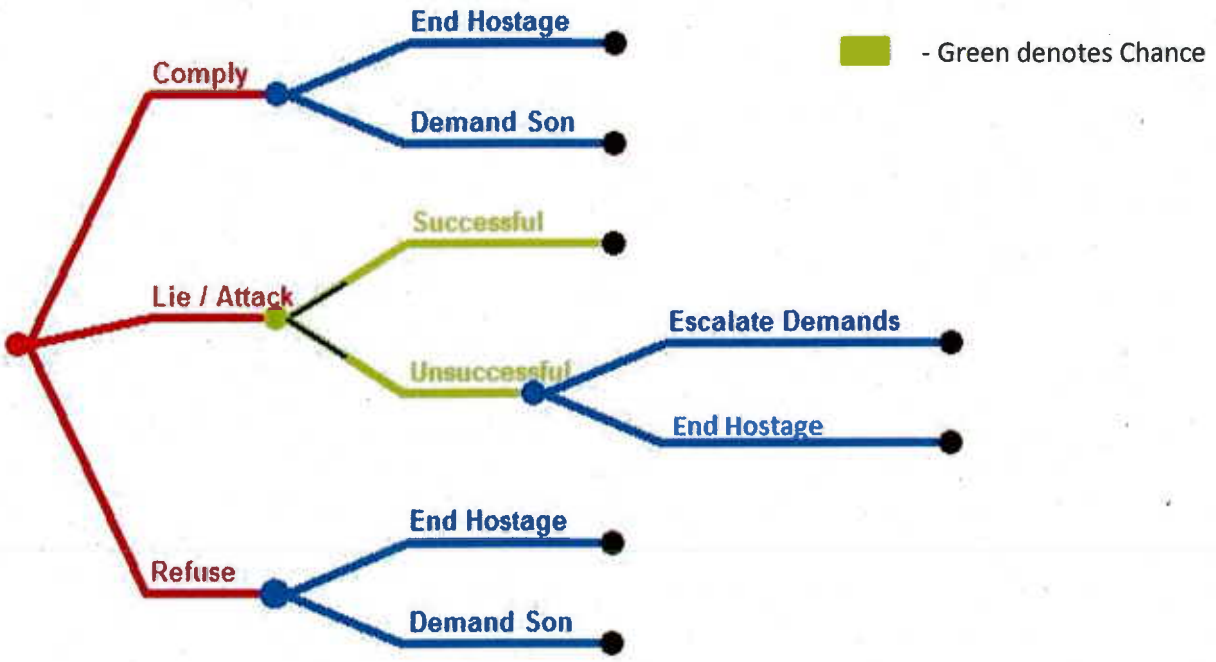
To start off it is important to examine what happens in the movie, John Q. The entire movie will not be reviewed, but rather the focus will be on the elements that pertain specifically to the conflict being modeled. Summed up in one sentence, the movie is one giant hostage situation that is not really what one would expect. In fact, the man taking people hostage does not care about money, power, attacking people, or anything like that. All he wants is for his son to live. The move starts with John Q's son, Mike, falling down and being diagnosed with an abnormal heart. He is only given months, days, or possibly only hours to live. Without money to afford a transplant, John Q takes matters into his own hands. He takes over the ER where Mike is residing, taking all the people present hostage. The stage is set, the cops

have arrived, and the hostage situation to be modeled has started.

We are only about one-third of the way through the movie at this point. However, since a game tree is being used to model the decisions that the different characters make in the movie, it will be helpful to explain the movie as we progress through the game tree. At this point it is worth noting that I have ignored John Q's choice to either initiate the hostage situation or to watch Mike die. I felt that this improbable decision was not important in the model of the hostage situation. Plus, it would have made for a blockbuster dud. Thus, the strategic game tree starts with the cops. They have the choice to either stand down or go on the offensive. If they stand down, a negotiation like environment is likely to develop. On the other hand, if they go on the offensive John Q must either surrender or things could get ugly. Going on the offensive does ensure an end to the hostage situation, but at what expense? In the movie, the cops initially stand down and begin to talk to john Q about their options.

The next section of the game tree models John Q's first demand that his son be placed on the organ donor list. However, in the movie there is first talk about releasing some hostages in good faith. This gives John Q the choice to just make demands without showing any good faith to the cops or to release some hostages in order to "be on the good side" of the cops. In the movie, John Q releases some of the harder to serve patients who need outside help and then he demands that his son be placed on the list.

Now the game tree is split in half so to speak. The cops have the same options whether John Q releases hostages or not. They can comply with his demands, refuse his demands, or rather just go on the offensive. In the case of the movie, they lie about adding Mike to the list and try to take him while he is vulnerable when talking to his son on the phone. If instead they would decide to comply, John Q has the choice to either end the hostage situation or to make additional demands. If he makes further demands, we are back to the beginning of this negotiation phase. We will talk more in depth about this possible turnout later in the report. If they refuse to cooperate with John Q's demands, John Q can again make another demand or give up. What differentiates this scenario from complying is the resulting outcomes to be discussed later. The option of trying to eliminate the threat comes with an associated probability. Because they are not sure whether the plan will be successful, I have introduced chance into the game tree. If successful, the hostage situation is over. However, if they are to fail, like in the movie, the situation has most likely escalated for both parties involved.



We are now approaching the end of the game tree. In all three of the cases above, John Q is given the choice to end the hostage situation or to demand that his son to be given to him. Behind the support of most everyone in the movie except for the cops, he demands his son. At this point, the cops can either comply by giving him his son, in the likely outcome the hostage situation will be over, or refuse the demand. If they would choose to refuse the demand, it is unclear what the result would be. However, taking this option into account, John Q could continue the hostage situation or just end the hostage situation. The potential continuation of the game tree will be discussed later.

Since we now know the entirety of the game tree, it is important to look into the preferences of the players involved. First, let us look into the different preferences John Q might have. It is clear in the movie that John's main objective is to save his son. When raising money to get on the donor list does not work, he takes matters into his hands and starts the hostage situation. However, John is not a violent man. Instead he orders the doctors held hostage to serve the sick people in need. He even releases some hostages to ensure they will get the treatment that they need. Therefore, one of John's preferences is the hostages' well-being.

The question still remains as to what John is willing to do in order to save his son. Obviously it is not to hurt anyone else. Instead, as the movie progresses, he demands that his son be put on the donor list and eventually wants Mike to be brought to him. The first demand would allow the opportunity for Mike to get a heart from someone else. This increases his chances of survival but is not much of an insurance since his health is already deteriorating quickly. The second demand has a much higher insurance for Mike's wellbeing. However, it

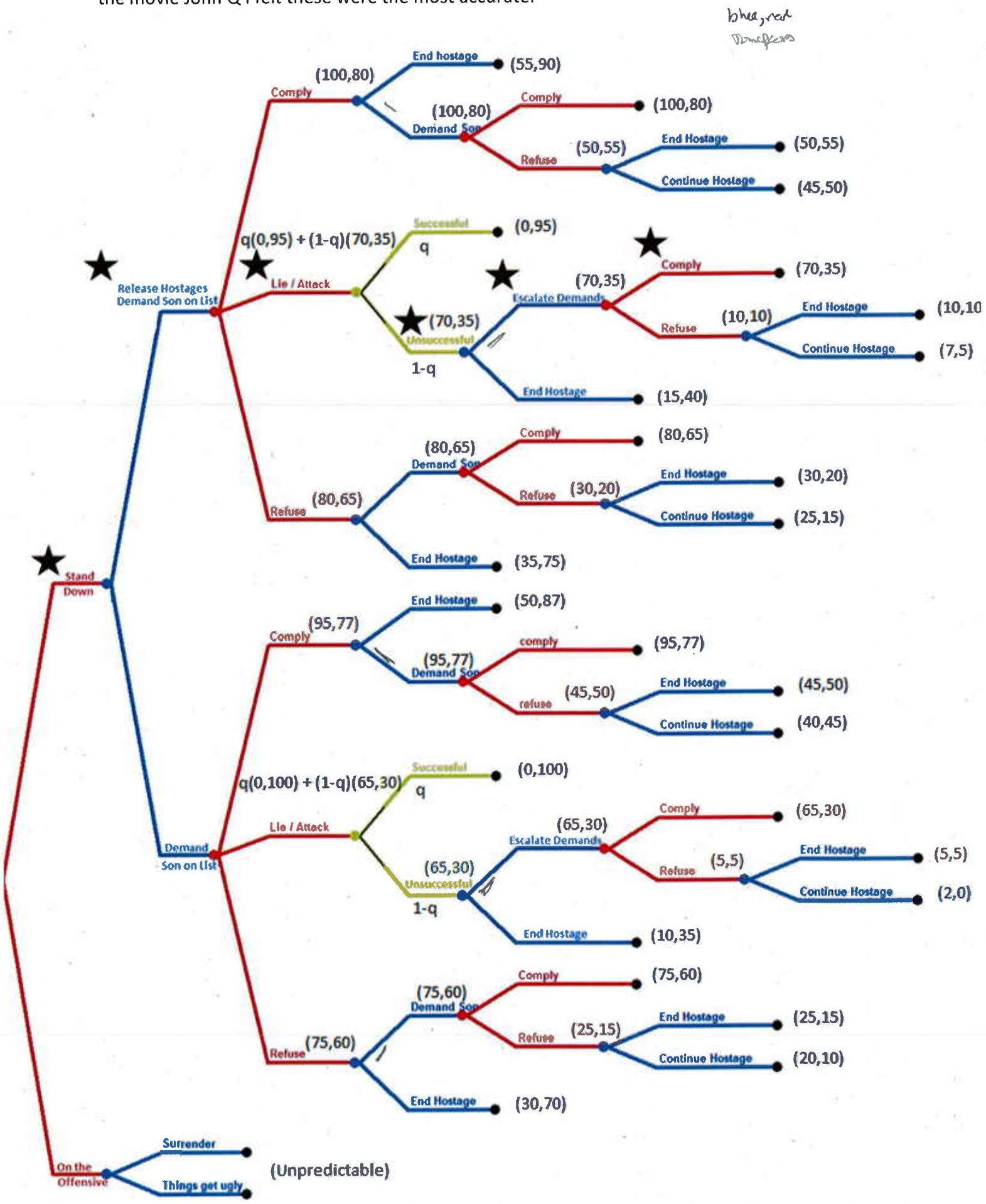
comes at an expense. John is willing to kill himself to save his son.

Now that we know something about John Q's preferences let us look at the preferences of the police. Obviously, the well-being of the hostages involved is the highest priority on their list. It is the job of the police to ensure the safety of innocent civilians and this would be a prime case. Additionally, the outside public looks more favorably on responses to hostage situations where nobody gets hurt, including the gunmen. Thus, the gunmen's health plays a role as well. However, as time progresses this preference begins to mean less and less.

Although safety is an important preference, other factors can play a big role. As shown in the movie, exciting conflicts like this situation very easily attract the attention of various news mediums. How the police handle a situation can easily shape the way society perceives the effectiveness of their police force. Thus, how long a negotiation lasts, the amount of publicity it attracts, and the appearance of how well they are doing can play a big part in how certain officers in charge might act. For example, the first negotiator was much more laid back and patient with John Q than the second negotiator. Thus, when time and appearance became an issue for the second negotiator, they decided to go on the offensive. The movie portrays the first negotiator disagreeing with this move or in other words, illustrating that he has different preferences.

For the game tree on the next page, I have chosen each player's preferences as follows. John Q's preferences go in this order: Saving his son, preserving the hostages' lives, and lastly his own life. I will go more into depth about the significance of these when I discuss outcomes. For the police: the hostage's lives, time/publicity/appearance, and finally the safety of the gunmen, John Q. Again these preferences can be changed to model a different scenario but for

the movie John Q I felt these were the most accurate.



As you can see, the outcomes are displayed on the previous page. These outcomes are based on the different preferences of each player. The highest outcomes for John Q existed when there was a high probability of Mike living. Thus, anywhere the police complied with bringing him his son resulted in a high payoff. The next highest contributor for John Q was getting his son's name on the list. Again, both of these factors contributed the most because John Q's top preference was saving his son. Since the safety of the hostages' was important, the part of the tree under refused had higher payoffs than those associated with the attacked section. This makes sense because being attacked puts you and others in immediate danger whereas not getting what you want just makes someone frustrated. Because John cared about his hostages, he has higher payoffs after releasing some hostages than if he did not. If it was his nature to be more aggressive and he did not care about the hostages then payoffs could be different. Another conclusion I made based on the movie was that if John did not get his son delivered to him, further demands would only decrease his chances of Mike being kept on the list. Therefore the payoffs for John diminish as he continues to prolong the hostage situation. For the most part, John Q's preferences are pretty straight forward.

For the police's outcomes it was important to distinguish what the outcome "end hostage" meant. This payoff is really a scale of the success or probability of success of the well-being of the hostages. If the payoff is high then John Q will likely walk away and everyone will be safe. However, if the payoff is low then the possibility of hurt hostages is much higher. As a result, deciding whether the hostages were safe or not played the largest role in deciding the outcomes. For example, complying with the gunmen will always create a safer environment for the hostages. The idea is that if the gunman is getting what he wants then he will not do

anything rash which could put someone in danger. For this reason, if the police decide to attack and do not succeed, their outcomes will be much less due to the element of danger. This is also the reason the police do not initially attack. They are unaware of John Q's behavior so they have no way of knowing what the outcome will be. Thus, I put unpredictable for these outcomes.

The second key element contributing to their payoffs is their preferences time/publicity/appearance and the safety of the gunman. These preferences are essential for determining at what probability of success the police are willing to attack. This is so important because the safest environment for the hostages is one without a gunman; in other words, the highest outcome is the one where they successfully eliminate their target. This makes sense because eventually the police will get tired of negotiating and will attack. Once an opportunity with a high enough outcome appears they will seize it.

Along with the payoffs, the backward induction is also shown on the game tree. However, it is not filled out past the police's decisions to comply, attack, or to simply refuse John's demand for his son to be placed on the list. The reason I stopped here is because we are unaware of the payoff if they should decide to attack. In other words, which choice would the police make between the payoffs: comply (100,80), Lie/Attack q(0,95) + (1-q)(70,35), and refuse (80,65). It is clear that the police would certainly comply over refuse because 80 is a higher payoff than 65. However, at what point is $80 > q(95) + (1-q)(35)$ or $q(95) + (1-q)(35) < 80$. To find this point we solve for q:

$$80 = q(95) + (1-q)(35)$$
$$80 = 95q - 35q + 35$$
$$q = .75$$

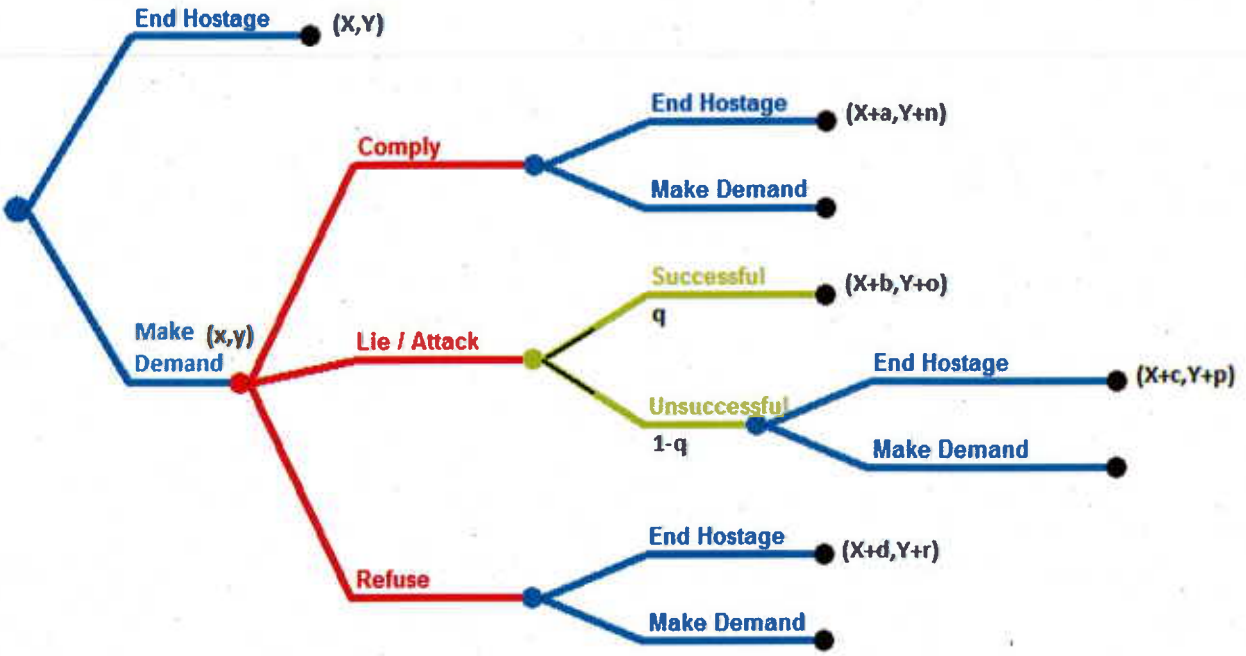
So if $q \leq .75$ then $80 > q(95) + (1-q)(35)$ and the police will comply with John's demand and if $q > .75$ then $q(95) + (1-q)(35) < 80$ and the police will decide to attack. Therefore, using these payoffs, the probability in the movie for eliminating John Q must have been greater than seventy-five percent.

At this point we have completed the model for the movie John Q. I have presented a game tree of all the different decisions made in the movie. The preferences of John Q and the police determined the many payoffs possible and their outcomes. By backward induction, q was found to be greater than .75 for the scenario depicted in John Q to unfold. So John narrowly escapes death, winds up getting his son's name on the list, and even witnesses the doctors putting a new heart in Mike. However it does not come without the expense of almost getting shot by a cop, coming about as close to suicide as possible, and after it is all said and done, going to prison.

Although the movie was obviously scripted to add as much drama as possible, it does produce some interesting ideas. How do you conduct backward induction with an infinite game tree where john keeps making demands? What would have happened if the first negotiator was the only one on the scene? Would have the movie John Q been so dramatic? These ideas will be looked at a little more in the next part of the report.

In the game tree of John Q, John Q started by making a demand for his son to be put on the donor list. This gave the police the decision to comply, Lie/Attack, or refuse. Depending on what they did, eventually John had to make another decision, make another demand or end the hostage situation. After looking at this situation, what happens if John keeps making demands? Does the game tree ever stop and at what point? In order to analyze this scenario I had to

simplify it as much as possible. I did this by making the following game tree with possible outcomes shown. The (X,Y) payoff refers to the payoffs each player would receive if John ended the hostage scenario instead of making another demand. If John did decide to make another demand and the police responded with one of its three options, John would again be back to the decision to end it or continue. However, the payoffs would be different. This change in possible payoffs is shown in the tree below with their different descriptions for the change. Also to note, (x,y) is the backward induction payoff that John must compare to (X,Y).



a – The increase in payoff due to a demand being accepted, positive but decreases as time increases

b – Consequence of getting eliminated, large negative number

c – You survived the attack but are in worse shape, negative number

d – Not as likely to get compliance after a refusal

Summary: a > 0 > d > c > b

n – Negative impact of time/publicity/appearance

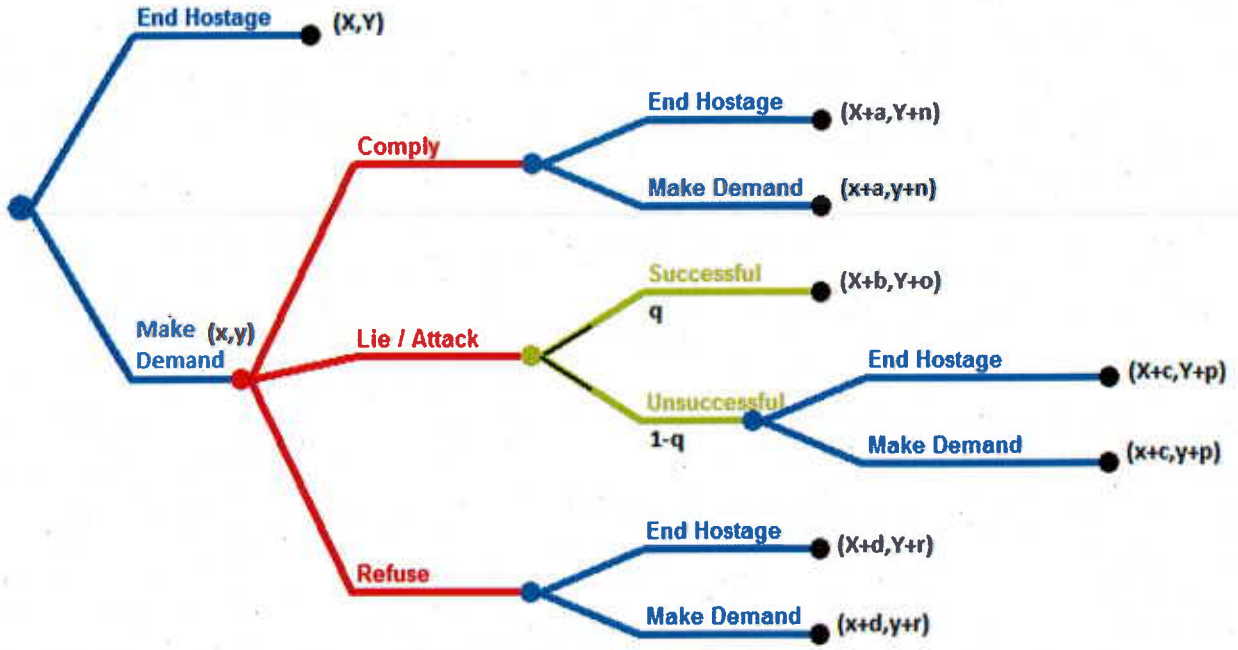
o – Proportional to preference for gunman's life, more the police care about his life => smaller o, increases with time

p – Major increased threat of negative outcome due to attempted elimination of target, large negative number

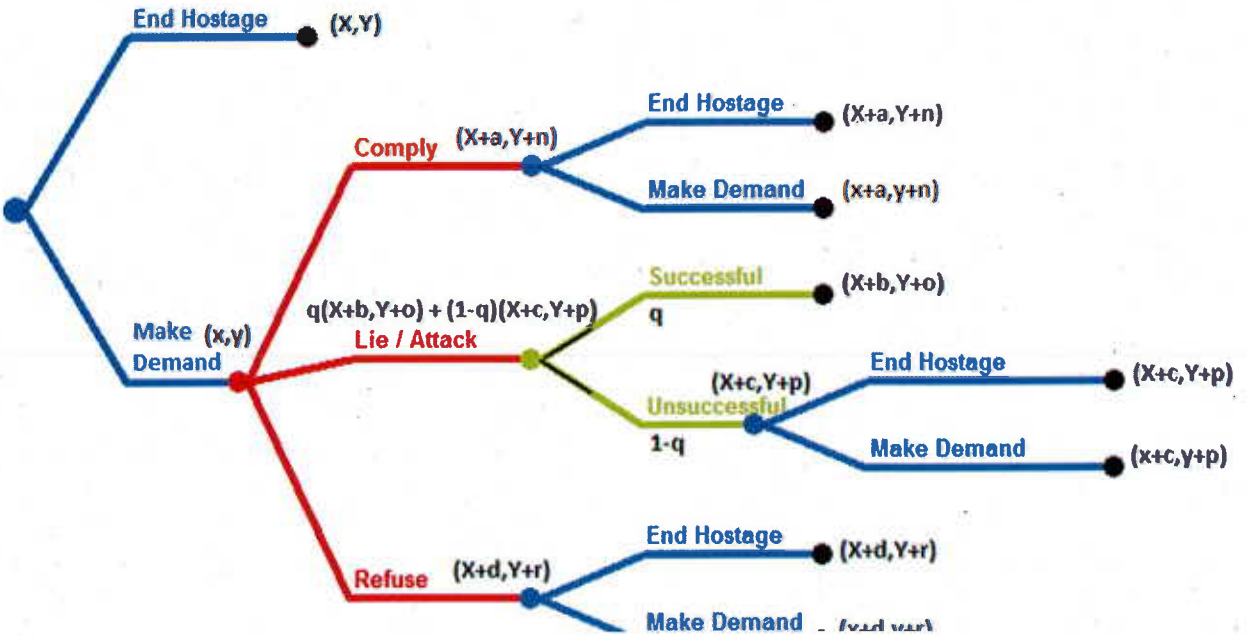
r – Minor increased threat of negative outcome due to refusal, negative number

Summary: o > 0 > n > r > p

The next step is showing the recurrence. As you can see, the game tree starts with John Q's decision to stop or make a demand and ends with the same decision. Thus, the game tree starts over again except (X,Y) are different. This is not a problem, the new game tree just starts with (X+its parameter, Y+its parameter). As a result, (x,y) is also changed to (x+its parameter, y+its parameter). These changes are shown below.



This now leaves us with two possible game trees, one in which (x,y) is a smaller payoff than (X,Y). Now that we have a finite tree, we can do the backward induction, shown below.



As before we are left with three options for payoff but are unsure which is the greatest. However we do know that the police would not choose to refuse over comply because Y+n ? Y+r simplifies to n ? r and we know (by the designated parameters) that n is always greater than r. This makes logical sense for the fact that the police would not intentially put hostages in danger without any possible gain out of the situation. Thus we know the decision comes down to whether Y+n is greater than q(Y+o) + (1-q)(Y+p). In either case, whichever is bigger will then be equal to (x,y). Therefore, in the case Y+n is greater, (X,Y) must be greater than (x,y) which is equal to (X+a,Y+n). This is never true since adding a positive number to X will always be larger than X. Therefore, if (X,Y) is larger than (x,y) then the backward induction must come from q(X+b,Y+o) + (1-q)(X+c,Y+p). This means that the probability of success for the cops in attacking is high enough. Solving for this q:

$(X+a,Y+n) < q(X+b,Y+o) + (1-q)(X+c,Y+p)$.

$Y + n < q(Y+o) + (1-q)(Y+p)$

$Y + n < qY + qo + Y + p - qY - qp$

$n < q(o-p) + p$

$$\frac{n-p}{o-p} < q$$

We must check first whether (x,y) is less than (X,Y):

(X,Y) > (x,y) is equal to q(X+b,Y+o) + (1-q)(X+c,Y+p).

$X > q(X+b) + (1-q)(X+c)$

$X > qX + qb + X + c - qX - qc$

$0 > q(b - c) + c$ and since b < c and c is negative then this is always true

We have shown that for all probabilities of success, (x,y) is less than (X,Y). Therefore, if $\frac{n-p}{o-p} < q$ then the police will choose to attack thus John Q would have no reason to make any more demands. In other words, if q reaches this point then John Q will end the hostage. This tells us a couple of things based on the parameters n, o, and p. If time/publicity/appearance is not a big deal then n will be small and the probability for success must be larger. There are two scenarios in which o will be a factor. Since the parameter o is based on how much the police favor the gunman's life, if initially they do not care then the probability of success does not have to be as high. Coupled to this is the fact that o is increasing as time progresses. Therefore, as time goes by the police are more likely to attempt to eliminate the gunmen. The last factor p is mainly dependent on how aggressive a gunman the police are dealing with. If the absolute value of p is larger, more aggressive gunman, then a smaller probability of success is needed. Basically, if the person is highly aggressive then they will be more likely met with gun shots opposed to compliance.

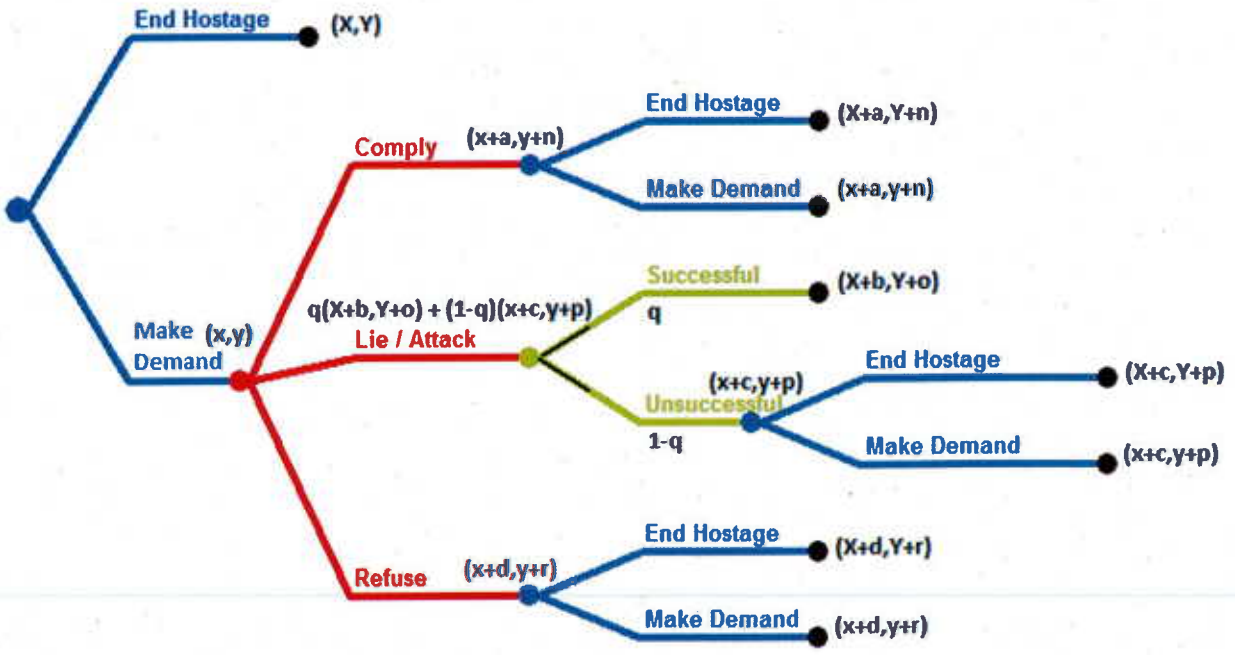
Now we must look at the game tree if (x,y) is larger than (X,Y). In this case, the backward induction is shown below. We know from earlier that complying gives a greater payoff than refusing so the cops would not choose to refuse. Consequently, (x+a,y+n) must be the backward induction payoff or q(X+b,Y+o) + (1-q)(x+c,y+p). We can rule out complying, (x+a,y+n), because x+a does not equal x for any values since a is negative. Therefore, our only option is if q(X+b,Y+o) + (1-q)(x+c,y+p) equals (x,y).

(x,y) = q(X+b,Y+o) + (1-q)(X+c,Y+p).

x = q(X+b) + (1-q)(x+c)

$\ast$ = qX + qb + $\ast$ + c – qx - qc

-c = q(X-x+b-c), since –c is always positive and (X-x+b-c) is always negative, this is never true. Therefore, (x,y) cannot be larger than (X,Y).



Now that we have looked more in depth about what factors into whether John Q should end the hostage or continue demanding, let us apply it to the movie. In the movie there were two negotiators for the police. One, the captain and second negotiator, had a lot higher preference about time/publicity/appearance and as a result had a smaller n parameter. He also seemed to care less about John Q so would have had a larger o parameter as well. Both these parameters lower the required probability of success needed to initiate an attack on the gunman. Consequently, in the movie the police took a shot at john Q pretty early on. However, if the original negotiator would have been still in charge, they would have been far less likely to take that route. So maybe we can credit the success and suspense of the film to the fact that they introduced a hotshot captain into the film that stirred things up. Otherwise the movie would have been pretty uneventful. It makes you wonder whether game theorists wrote the script.